Neutrino Oscillations through Entanglement

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Despite the fact that neutrino oscillations have been theoretically and experimentally studied over the past half-century, the "right" way for deriving neutrino oscillations is a topic of much debate. Recently, physicists have formulated a new method of deriving neutrino oscillations starting with an entangled neutrino and recoil partner created through two-body decay. The advantages of this method are that energy and momentum conservation is manifest in the parent rest frame and neutrino oscillations are derived without explicitly defining a source or detector.

I. AN INTRODUCTION TO NEUTRINOS AND THEIR OSCILLATIONS

Neutrinos, the "little neutral ones", are sometimes called "ghost" particles. They have no charge, very little mass compared to other particles, and only interact with the weak force and a bit through gravity. Neutrinos are difficult to detect because of these properties. There are millions of neutrinos in every cubic centimeter of the universe but a single neutrino can pass through several lightyears of lead without being absorbed. Neutrinos play an important role in countless interactions such as radioactive decay and fusion.[9] But the most mysterious aspect of neutrinos is that there are at least three types of neutrinos and they can change into each other while simply moving through space.

The Standard Model contains three neutrinos, the electron neutrino (ν_e), the muon neutrino (ν_μ) and the tau neutrino (ν_τ). Each neutrino is named after the charged lepton with which they are produced in weak interactions, such as radioactive decay. In the Standard Model, these neutrinos are massless and left-handed, meaning their spin is always anti-aligned with their momentum.

However, contrary to the Standard Model, neutrinos have mass. Neutrino mass was theoretically postulated in the 1970s and experimentally confirmed in the 1990s. But these mass eigenstates, ν_1, ν_2, and ν_3, are not the same as the weak (or "flavor") eigenstates, ν_e, ν_μ, and ν_τ. Instead, each of the weak eigenstates is a superposition of the mass eigenstates. The significance of this mixing of flavor and mass eigenstates is the weak eigenstates evolve into each other. This means a ν_e can evolve into a ν_μ and likewise for the other combinations of the three weak eigenstates. This phenomenon is called neutrino oscillation. Neutrino oscillations are interference of the mass eigenstates that comprise a weak eigenstate. It is by observing neutrino oscillations (measuring the relative appearance and disappearance of the three neutrino weak eigenstates as neutrinos propagate in space) that we know neutrinos have mass.

I.1. The Mixing Matrix

Neutrino weak eigenstates are a superposition of mass eigenstates.

$$|\nu_\alpha\rangle = \sum_i c_{\alpha i}|\nu_i\rangle$$

The coefficients, $c_{\alpha i}$, of this superposition are contained in the neutrino mixing matrix U. The neutrino mixing matrix is a 3x3 rotation matrix that translates between the weak and mass eigenstates bases.

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$, where θ_{ij} are the mixing angles [4]. The variables δ and $\alpha_{1,2}$ are phases that modify the matrix depending on whether or not right-handed neutrinos exist [5],[9]. These phases do not alter the derivation of neutrino oscillations but do effect mixing and the resulting probabilities of oscillation.

To simplify calculations in this paper, we will consider a two-neutrino model. This does not alter our derivation of neutrino oscillations. Our two-neutrino system is described by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{2}$$

where θ is the only mixing angle and ν_1 and ν_2 are mass eigenstates with non-degenerate masses.

I.2. Oscillations and Experiments

Neutrino oscillations are generated by the interference of mass eigenstates upon the detection of a weak eigenstate. We will show that wavelength for these oscillations is

$$\lambda_{i,j} = \frac{4\pi E}{\Delta m^2_{i,j}} \ , \tag{3}$$

Where we are using units $\hbar = c = 1$.

Neutrino oscillation experiments measure the relative appearance or disappearance of weak eigenstates. This is done by first determining the production rate of each flavor eigenstate at the source of the neutrinos (either a natural source, reactor, or a accelerator beam) and then comparing the ratios at a set distance away within the oscillation wavelength. Experimentalists may control how long of a oscillation wavelength they wish to study by controlling the energy of the neutrinos of their source.

I.3. Neutrino Masses and Mixing Angles

Current experiments have put the following bounds on neutrino mass differences, mixing angles, and maximum neutrino mass [4].

$$\sin^2(2\theta_{12}) = 0.83 \pm 0.03$$
$$\sin^2(2\theta_{23}) > 0.92$$
$$\sin^2(2\theta_{13}) < 0.15$$

$$\Delta m^2_{21} = (7.59 \pm 0.20)10^{-5}\ eV^2$$
$$\Delta m^2_{31} \approx \Delta m^2_{32} = (2.43 \pm 0.13)10^{-3}\ eV^2$$
$$m < 2\ eV$$

Measuring the actual masses of neutrinos is tricky because (1) neutrino masses are very small (2) neutrinos only enter interactions via the weak force and thus as weak eigenstates. Measurements of neutrino masses are made by studying beta decay ($n \to p + e^- + \bar{\nu}_e$). The energy spectrum of the three body decay gives limits on the neutrino masses [9].

I.4. Notation: Indicies and Units

Throughout the derivation, the subscripts α and β on mixing matrix and neutrino states refer to weak eigenstates and indices i and j refer to mass eigenstates.

Unless stated otherwise, we use units where $\hbar = c = 1$.

II. DERIVING OSCILLATIONS

Now the we have presented the basic properties of neutrinos, we will show how and why neutrinos oscillate.

There are many ways to derive neutrino oscillations.[1, 5, 9] The "right" way to derive neutrino oscillations is a topic of much debate. Even though neutrino oscillations were theoretically derived 40 years ago and first experimentally observed 20 years, neutrino oscillations are still an active research topic. There are many subtleties to the calculation of neutrino oscillations and several different assumptions one can make to get to the standard expression for neutrino oscillations.

The derivation of neutrino oscillations starting with an entangled neutrino utilizes some topics that are a bit beyond 8.06 (such as density matrices for entangled states), but mostly requires a solid understanding of undergraduate quantum mechanics. Though this derivation, we will illustrate a few aspects of neutrino oscillations beyond just deriving the standard wavelength of oscillation.

Most often, neutrino oscillations are derived using a wave packet in an initial weak eigenstate. The weak eigenstate, composed of the three mass eigenstates, is localized within the width of the packet. In some derivations, it is assumed that the superposition of mass eigenstates constructing the weak eigenstate have either the same energy or the same momentum. One way to think of this is, neutrinos come from the same beam or source and therefore have the same energy. Alternatively, a neutrino wavepacket propagates in space without decohering therefore the mass eigenstates must have the same momentum. While these assumptions simplify calculation and give the experimentally correct phases for neutrino oscillation, these assumptions are incorrect.

These assumptions are incorrect because a neutrino is created in a weak eigenstate through an interaction that must conserve energy and momentum. Energy and momentum conservation requires each mass eigenstate carry a different energy and momentum. There is no frame we can Lorentz transform to in which all three energies or all three momenta are equal. Neutrino oscillations can be correctly derived with wave packets without the equal-energy or equal-momenta assumption, but the derivation is more technically difficult [1].

In this paper, we present a derivation of neutrino oscillations that employs entanglement to simplify the calculation [2, 3, 7]. In our case, a parent particle decays into a neutrino and recoil partner. The subtlety in this approach is that as long as the neutrino is entangled with another particle, the neutrino will not oscillate into different weak eigenstates. However, neutrinos do oscillate once the particles become disentangled. Once the particles become disentangled, they can be described independently. A measurement of one will no longer imply anything about the state of the other. We will show this mathematically through the use of density matrices (following the derivation of reference [2]). We will also examine the different conditions under which neutrinos oscillate, and why we detect oscillations experimentally.

II.1. Entanglement in Two-Body Decay

Consider a weak eigenstate neutrino created through two-body decay:

$$P \to \nu_\alpha + R \tag{4}$$

The neutrino ν_α and the recoil parter R are entangled. An example of this is charged pion decay ($\pi^+ \to \mu^+ + \nu_\mu$).

This means, the wavefunction describing the two particles cannot be expressed as a tensor product of states. In this case, measuring the energy and momentum of one particle determines the energy and momentum of the other. However, a neutrino in a weak eigenstate is a superposition of mass eigenstates, therefore the wavefunction of the weak eigenstate neutrino and recoil particle is a superposition of entangled states each satisfying energy and momentum conservation. Entanglement is a fundamental feature of neutrino production that affects how the neutrino propagates through space.

II.2. Deriving Neutrino Oscillations though Entanglement

We can write the entangled wavefunction in terms of the mass eigenstates.

$$|\psi\rangle = \sum_i U^*_{\alpha i}|R\,(k_i)\nu_i\,(q_i)\rangle$$

where ν_i represents one of the mass eigenstates ν_1, ν_2, or ν_3 and $U_{\alpha i}$ is lepton mixing matrix from Eqn. 1. k_i and q_i are the energy and momentum vector of the recoil partner and the neutrino mass eigenstate. Note that the subscript i on the momentum of the recoil particle indicates it is dependent on the mass eigenstate with which the recoil particle is entangled. Also note that we are assuming our neutrinos move in one spatial dimension and one time dimension. We would otherwise need to integrate over all possible directions for the momenta in three dimensions and integrate over phase space. Thus k_i and q_i are the "Two-momentum" vectors of the neutrino and recoil particle. This is a reasonable assumption since neutrinos are only detected in experiments if they move from the source to the detector, in a straight line.

In a two-neutrino model, Eqn. 5 simplifies to

$$|\psi\rangle = \frac{1}{\sqrt{N}}[\cos(\theta)|R(k_1)\nu(q_1)\rangle + \sin(\theta)|R(k_2)\nu(q_2)\rangle] \quad (5)$$

where we have made the assumption that our neutrino begins its life in the electron flavor eigenstate. N is the normalization factor.

Now that we have written down the state of our neutrino and recoil particle, we want to determine whether we see neutrino oscillations while the recoil particle and the neutrino are entangled. To do this, we construct the density matrix of our state.

II.2.1. The Density Matrix

The density matrix is a way of representing ensembles of quantum systems, versus as the state vector $|\psi\rangle$ is a representation of a quantum system. The density matrix contains more information than the state vector (accounting for mixed quantum systems) and using a density matrix allows us to simplify our entangled system and inherently see whether there is interference.

The density matrix of a state is defined as

$$\rho \equiv \sum_i p_i|\psi_i\rangle\langle\psi_i|$$

where p_i is the probability of a system being in state $|\psi_i\rangle$. For a "mixed" state, a statistical ensemble of quantum systems, the density matrix is composed of the sum of the projection matrices of each possible state multiplied by the probability of measuring that state. An example of a mixed state is an ensemble of N particles, each of which is in an energy eigenstate. The entire system can be described as a sum of the energy eigenstates in the system multiplied by coefficients, the modulus squared of which gives the probability of measuring a particle with that energy.

A "pure" state, on the other hand, is simply a quantum system. The state in Eqn. 5 is entangled and thus a pure state. The density matrix of a "pure" state is simply the projection matrix of a state [6, 8]. Let $P_{R\nu_{i,j}} \equiv |R_i\nu_i(q_i)\rangle\langle R_j\nu_j(q_j)|$. The density matrix for the state in Eqn. 5 is

$$\begin{aligned}\rho_{R\nu} &= \frac{1}{\sqrt{N}}|\psi\rangle\langle\psi| \\ &= \frac{1}{\sqrt{N}}[\cos^2(\theta)P_{R\nu_{1,1}} + \cos(\theta)\sin(\theta)P_{R\nu_{1,2}} \\ &\quad + \cos(\theta)\sin(\theta)P_{R\nu_{2,1}} + \sin^2(\theta)P_{R\nu_{2,2}}]\,.\end{aligned}$$

In matrix form,

$$\rho_{R\nu} = \frac{1}{\sqrt{N}}\begin{bmatrix}\cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin^2(\theta)\end{bmatrix}.$$

This matrix is still in terms of entangled states, but we are interested in neutrino oscillations, which do not involve the recoil particle. Our system of entangled states is composed of two subsystems: that of the neutrino and that of the recoil particle. The density matrix of the neutrino is given by the partial trace over the degrees of freedom of the recoil particle:

$$\begin{aligned}\rho_\nu &= tr_R(\rho_{R\nu}) \\ &= \sum_{i,j}\langle R(k_i)|R(k_j)\rangle|\nu_i(q_i)\rangle\langle\nu_j(q_j)|\,.\end{aligned}$$

This operation is equivalent to assuming no knowledge of the recoil partner state. This is one of the most powerful applications of density matrices techniques, especially when dealing with entangled states.

Let $\eta_{R_{i,j}} \equiv \langle R(k_i)|R(k_j)\rangle$. The resulting neutrino density matrix is

$$\rho_\nu = \frac{1}{\sqrt{N}}[\cos^2(\theta)\eta_{R_{1,1}}P_{\nu_{1,1}} + \cos(\theta)\sin(\theta)\eta_{R_{1,2}}P_{\nu_{1,2}} + \cos(\theta)\sin(\theta)\eta_{R_{2,1}}P_{\nu_{2,1}} + \sin^2(\theta)\eta_{R_{2,2}}P_{\nu_{2,2}}]$$

where $\eta_{R_{1,1}} = \eta_{R_{2,2}} = 1$. In matrix form,

$$\rho_\nu = \frac{1}{\sqrt{N}}\begin{bmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta)\eta_{R_{1,2}} \\ \cos(\theta)\sin(\theta)\eta_{R_{2,1}} & \sin^2(\theta) \end{bmatrix} \quad (6)$$

The diagonal terms of the matrix describe the probability of the recoil partner interacting with each neutrino mass eigenstates. The off-diagonal terms describe the interference of the mass eigenstates states. The off-diagonal terms measure the "coherence" of the subsystem. Neutrino oscillations arise from interference of the mass eigenstates, so neutrino oscillations occur only if the off-diagonal terms are non-zero.

We can simplify this matrix by employing the fact that the momentum of the recoil particle is dependent on the neutrino mass eigenstate with which it interacts. Thus $\eta_{R_2,R_1} = \eta_{R_2,R_1} = 0$. The density matrix for the entangled neutrino is

$$\rho_\nu = \frac{1}{\sqrt{N}}\begin{bmatrix} \cos^2(\theta) & 0 \\ 0 & \sin^2(\theta) \end{bmatrix} . \quad (7)$$

In this case, the off-diagonal terms are zero, so there are no neutrino oscillations. While the neutrino and recoil partner are entangled, the neutrino remains in its initial electron flavor eigenstate.

II.3. Obtaining Neutrino Oscillations from an Initially Entangled State

If entangled neutrinos cannot oscillate, the neutrino and recoil partner must become disentangled somehow. We will discuss two ways in which this can happen.

II.3.1. Measurement of the Recoil Particle and Neutrino and The Oscillation Wavelength

Consider an experiment in which both the recoil particle and the neutrino are measured. In neutrino experiments, the recoil particle is usually not measured, but this thought experiment will illustrate interesting properties of neutrino oscillations.

Suppose we measure the recoil particle to be in the state $|\bar{R}\rangle$. Making this measurement is equivalent to projecting $|\bar{R}\rangle$ on our original state $|\psi\rangle$.

$$\begin{aligned} |\bar{R}\rangle\langle\bar{R}|\psi\rangle &= \frac{1}{\sqrt{N}}|\bar{R}\rangle\langle\bar{R}|(\cos(\theta)|R(k_1)\nu(q_1)\rangle \\ &\quad + \sin(\theta)|R(k_2)\nu(q_2)\rangle) \\ &= \frac{1}{\sqrt{N}}[\eta_{\bar{R},R_1}\cos(\theta)|\bar{R}\rangle|\nu(q_1)\rangle \\ &\quad + \eta_{\bar{R},R_2}\sin(\theta)|\bar{R}\rangle|\nu(q_2)\rangle] \end{aligned}$$

The state is now disentangled. The wavefunction for our neutrino becomes

$$|\psi_\nu\rangle = \frac{1}{\sqrt{N}}\left[\eta_{\bar{R},R_1}\cos(\theta)|\nu(q_1)\rangle + \eta_{\bar{R},R_2}\sin(\theta)|\nu(q_2)\rangle\right] .$$

Our mass eigenstates are momentum and energy eigenstates, so we can write them explicitly as plane waves with term $e^{iq\cdot x}$, where x is the two-velocity. Furthermore, we can express our wavefunction in terms of flavor eigenstates by using the inverse of the two neutrino mixing matrix (2).

$$|\psi_\nu\rangle = \frac{1}{\sqrt{N}}[\eta_{\bar{R},R_1}\cos(\theta)e^{iq_1\cdot x}\left(\cos(\theta)|\nu_e\rangle - \sin(\theta)|\nu_\mu\rangle\right) + \eta_{\bar{R},R_2}\sin(\theta)e^{iq_2\cdot x}\left(\sin(\theta)|\nu_e\rangle + \cos(\theta)|\nu_\mu\rangle\right)]$$

Therefore the amplitude for measuring the electron flavor eigenstate is

$$A = \frac{1}{\sqrt{N}}[\eta_{\bar{R},R_1}\cos^2(\theta)e^{iq_1\cdot x} + \eta_{\bar{R},R_2}\sin^2(\theta)e^{iq_2\cdot x}] \quad (8)$$

and the probability

$$P_{e\to e} = \frac{1}{N}[\eta^2_{\bar{R},R_1}\cos^4(\theta) + \eta^2_{\bar{R},R_1}\sin^4(\theta) + 2\eta_{\bar{R},R_1}\eta_{\bar{R},R_2}\sin^2(\theta)\cos^2(\theta)\cos((q_1 - q_2)\cdot x)] \quad (9)$$

The probability has a phase $\phi = (q_1 - q_2)\cdot x$ as long as $\eta_{\bar{R},R_1}$ and $\eta_{\bar{R},R_2}$ are non-zero. ϕ is the phase of the neutrino oscillation.

The mass eigenstates spatially separate as they travel due to their difference in momenta. Oscillations only occur under the condition that the wavefunctions of the mass eigenstates overlap sufficiently to produce interference upon detection. If we think in terms of wave packets, this means that the separation between mass eigenstates cannot be greater than the width of the localization of the wave packet, $v\Delta T$:

$$(v_1 - v_2)t << \frac{(v_1 + v_2)}{2}\Delta T \quad (10)$$

where v is the average of the velocities of different neutrino mass eigenstate components of the wave packet. If this condition is not satisfied, neutrino oscillations "wash out" and the oscillation term averages to zero.

Assuming this condition is satisfied, we can interpret the two-vector x so that we can express these oscillations in terms of a meaningful wavelength. At the time of detection, $x = (t, d)$ (when and where the neutrino is detected). For the neutrino wavepacket $d \approx vt$ where $v = \frac{p}{E}$ the fraction of the average momenta over the average energy of the neutrino mass eigenstates. Using this,

$$x = (t, vt) \approx t(1, \frac{p}{E}) = \frac{t}{2E}(q_1 + q_2) \tag{11}$$

where q_1 and q_2 are the same "two-momentum" vectors used earlier, the time component of which is approximately equal to the average energy, canceling the average energy denominator, recovering t. The phase ϕ becomes

$$\phi = (q_1 - q_2) \cdot x = \frac{t}{2E}(q_1 - q_2) \cdot (q_1 + q_2) = \frac{t\Delta m_{1,2}^2}{2E} \tag{12}$$

where, in the case of relativistic neutrinos, $t \approx d$. Thus the oscillation wavelength, $\lambda_{1,2}$ is

$$\lambda_{1,2} = \frac{4\pi E}{\Delta m_{1,2}^2} \tag{13}$$

which agrees with the standard and experimentally verified expression for neutrino oscillations [5, 9].

Now that we have defined the conditions for oscillations and derived the oscillation wavelength, we return to discussing our measurement of the recoil partner. We have two cases. Either $\eta_{\bar{R},R_1}$ and $\eta_{\bar{R},R_1}$ are both nonzero or one is zero. Both cannot be zero because the recoil partner must be measured to have a momentum contained in our initial entangled state. In the first case, neutrinos will oscillate with the wavelength $\lambda_{1,2}$. The second case is more subtle.

Suppose we measure the momentum of the recoil partner. Assuming that we have an errorless measurement device, the precision of the measurement depends on the degree to which we localize the recoil particle in space in the process of measuring it, due to quantum uncertainty.

If we measure the momentum without localizing the particle in space, physically impossible but theoretically interesting, we would measure the particle to have a momentum of either k_1 or k_2. In doing so we would be able to determine which mass eigenstate interacted with recoil partner. This collapses the neutrino wavefunction down to a single mass eigenstate. No oscillation would occur but we would still be able to detect a flavor change by detecting the neutrino since the mass eigenstate is itself a superposition of flavor eigenstates. The probability of measuring it in the electron flavor state becomes $\cos^4(\theta)$ and in the muon flavor, $\sin^4(\theta)$.

We can obtain the same result by considering the uncertainty in the location of the neutrino. Due to entanglement, measurement of the recoil particle's momentum determines the momentum of the neutrino mass eigenstate with which the recoil interacted. Since the momentum of the mass eigenstate is known, the particle cannot be localized in space. Thus the uncertainty Δx is essentially infinite and the condition in Eqn. 10 is violated. The $v\Delta T$ goes to infinity since a plane wave is essentially a wave packet with infinite width. Also, the decay point is unknown due to Δx and $t \approx d$, the left hand side of the condition $(v_1 - v_2)t$ goes to infinty. Thus the two sides become comparable. The separation of the mass eigenstates is no longer negligible and the neutrino does not oscillate. The oscillation component of the probabilities is averaged out to be zero [5].

II.3.2. Neutrino Creation in Real Neutrino Experiments

So far we have considered examples where both the recoil particle and the neutrino are measured. As mentioned before, in actual neutrino experiments the recoil particle is not usually measured, so how do oscillations arise? Some suggest that the recoil interacting with the surround environment causes disentanglement, [10]. But there is another way in which the neutrino can be disentangled from its recoil partner.

At the beginning of our derivation we made one assumption by boosting to the rest frame of the parent particle. In order to do this we assumed is the parent particle was in a energy and momentum eigenstate. This is a valid assumption as long as the parent particle is not localized and we are completely uncertain of its position in space. This is not the case is an experiment. In most experiments, we measure neutrinos from a distinct source: the sun, a beam, or a reactor. This gives us a bound on the location of the creation of the neutrino. Because of this, the wavefunction of parent particle in momentum space is a distribution of momentum eigenstates. This uncertainty propagates to the parent's decay products.

Taking the spread of the parent particle momentum into account the density matrix Eqn. 6 for the entangled neutrino has non-zero off-diagonals. Therefore, neutrino oscillations occur despite the neutrino being entangled. At first, we might think that this contradicts our previous results, but it is important to understand how the neutrino is entangled. The neutrino is still a superposition of product states, but those product states are not momentum eigenstates of a specific mass eigenstate. The neutrino mass momenta are nots entangled with the recoil momenta, therefore interference due to mass differences can arise. In order for these off-diagonal terms to be non-zero, the width in the momentum only needs to be on the order of 10^{-10} eV, meaning that the parent particle is localized within approximately 1 km. This is the case for most experiments using a terrestrial neutrino source such as a reactor or a beam.

III. ADVANTAGES AND SUBTLETIES OF THE ENTANGLEMENT DERIVATION

This derivation confirms the standard expressions for neutrino oscillations which agree with experiment. It demonstrates that neutrino oscillations only occur through disentanglement of the mass eigenstates and the recoil partner. The entanglement conditions imposed on the created neutrino reduced the number of additional assumptions needed to solve for neutrino oscillations. The neutrino mass eigenstates and recoil partner are created under kinematic constraints imposed by two-body decay. Essentially, the only additional assumptions that we made were that the neutrino is detected at $x = (t, d)$ and that if mass eigenstates sufficiently separate as they propagate in space, there are no oscillations. [2]

This derivation also illustrates several properties of neutrino oscillations, when they will and will not oscillate due to entanglement and uncertainty. If a neutrino is sufficiently delocalized in space, it will not oscillate until an interaction with the recoil partner breaks its entanglement with its recoil partner. Once disentanglement occurs, oscillations only arise if the neutrino is detected at a distance comparable to the oscillation wavelength. Otherwise, the oscillations are "washed out" by the separation of the neutrino mass eigenstates. If a neutrino is entangled and sufficiently localized in space, oscillations arise because the superpositions that form the entangled state are not momentum eigenstates of a specific mass.

IV. NEUTRINO OSCILLATIONS AND NEW PHYSICS

Neutrino oscillations may lead physicists to physics beyond the standard model. Experimentally, deviations from standard oscillation probability expressions suggest CP violation in the neutrino sector (asymmetry between how neutrinos and anti-neutrinos interact). Some deviations also suggest that there may be sterile neutrinos, additional mass eigenstates that are not part of the weak eigenstates and thus do not interact via the weak force. [9]

The entanglement derivation of neutrino oscillations may explain some cases of these deviations. Reference [10] proposes that the time scales for disentanglement to occur may have an effect on the oscillations measured by short baseline neutrino experiments which employ detectors approximately one kilometer away from their source.

Despite the presence of neutrino oscillations in physics literature for the past half-century, neutrino oscillations are still an area of unresolved nuances. There are many ways to derive the standard results, but there is much debate of the "right" way to approach their derivation and predictions for oscillation conditions vary between derivations [1–3, 7, 10]. The quantum mechanics of neutrinos and their oscillations is still full of rich problems to be solved by the next generation of physicists.

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